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Module : Nuclear Models -1

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Description of Module	
Subject Name	Physics
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1. Nuclear Models

In order to understand the various observed properties of the nucleus of an atom it is important to have an adequate knowledge about the nature of the inter-nucleon interaction. In the nucleus three different kinds of force takes part (electromagnetic, weak, and strong forces), and there exist a very strong short range force between the nucleons. The combination of all the three forces makes the nuclear force quite complicated, yet to be fully understood, however, the exact mathematical form of this interaction is still not known.

So far none of the proposed theories gives us a full understanding of the nature of the inter-nucleon interaction. Even if the exact nature of the interaction were known, it would have been extremely difficult to develop a satisfactory theory of the structure of the nucleus made up of a large number of neutron and protons, since it quite impossible to solve the Schrödinger equation exactly for such a many body system. As the total wave function of the nucleus becomes too complicated to be useful for most such a many body system and it helps only in light nuclei. Various methods have been proposed to solve the problem with different degree of approximation. However, the problem is still far from being solved completely.

For this reason the analysis of the nuclear structure is more complex than the analysis of many electron-atoms. Because of these difficulties in developing a satisfactory theory and explaining of the nuclear structure, different nuclear modes have been proposed for the nucleus which are of great help, each with different level of success, in gaining some insight into nuclear structure. The nuclear models are the way of describing the observed nuclear properties using basic principles of science and that gives a physical insight into a wide range of its properties.

A successful model not only explains the existing data but also has the power to make predications; the model should also be able to explain experimental results successfully. Different approaches and different analogy gives different nuclear models.

1.1 Classification of Nuclear Models

Nuclear models are classified into two main groups. First group is the independent particle model (or individual particle model), in which there is little or no interaction take place between the individual particles that constitute nuclei, and each nucleons occupy discrete energy states. The Semi-classical Fermi gas model and quantum mechanical Shell model fall into this group. Second group is the Collective model with no individual particle states, in this model individual protons and neutrons are mutually coupled to each other. The semi-classical Liquid drop model and quantum mechanical rotational and vibrational model fall into group.

Nuclear Models

	Semi classical	**Quantum Mechanical**
Independent Particle	**Fermi Gas**	**Shell**
Collective	**Liquid Drop**	**Rotational** **Vibration**

2. Liquid Drop Model

The liquid drop model gives good account of bulk properties of nuclei. It was first proposed by the George Gamow in 1928, later expanded by N. Bohr and F. Kalckar in 1935 and was later applied by C.F. von Weizsacker and H.A. Bethe to develop a semi-empirical formula for the binding energy of the nucleus. It is based on the observation that the all nuclei have nearly same density.

In the liquid drop model nucleus is treats as a drop of incompressible nuclear liquid. As in the case of nucleus, nuclear properties like density and binding energy per nucleon is nearly constant for most of nuclei, it is same as in case of liquids where latent heat of vaporization is nearly proportional to the number of molecules within the liquid. This fact shows a close analogy of the nucleus with a liquid drop. Thus we conclude that the inter-nuclear force within the nucleus reach to a saturation value, so that each nucleon can interact only with its nearest neighbors as in case of liquids where each individual molecule within a liquid drop exerts an attractive force upon in its molecules in its immediate neighbourhood.

2.1 Basic Assumptions

While treating nucleus as a drop of liquid some assumptions were made which are given as follows:

- The force of attraction between the nuclear surfaces is similar to the force of surface tension on the surface of the liquid drop.
- As in the case of liquid drop the density of the nuclear matter is independent of its volume. As nuclear radius R~ $A^{1/3}$ where A is the mass number. Hence the nuclear volume V is proportional to A.
- The nuclear force saturates, i.e. each nucleon can interact only with nucleons in its close vicinity.

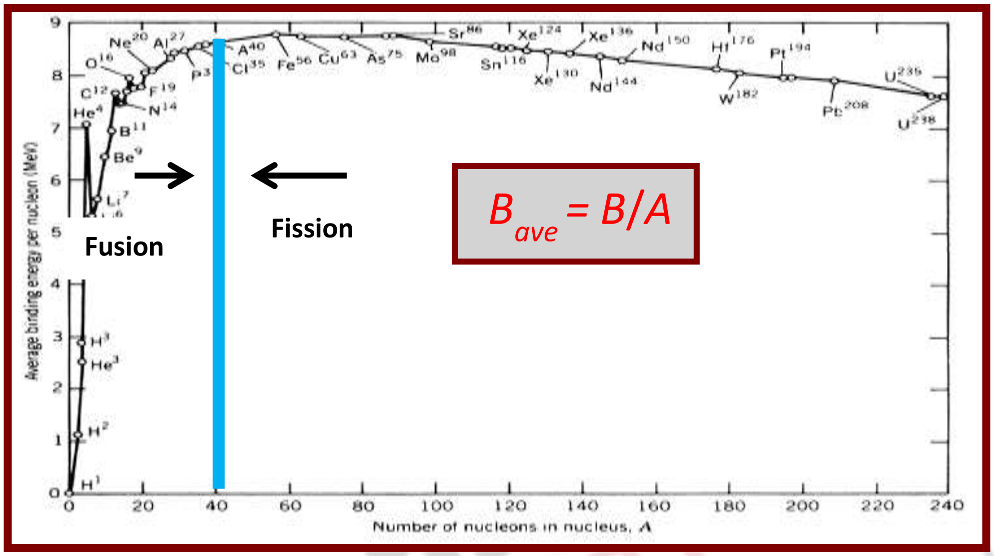


Fig.1: Graph showing variation of binding energy per nucleon with mass number A is roughly constant.

- The nuclear interactions between protons and protons, between protons and neutrons, and between neutrons and neutrons appears to be identical, so the nuclear force is identical for every nucleons, i.e., Vpp= Vpn = Vnn (where V denotes the nuclear potential).
- The mean free path of nucleons in nuclear matter is smaller than the nuclear diameter (as in liquids).
- The internal energy of the nucleus is analogous to the heat energy within the liquid drop.
- The formation of a short lived compound nucleus by the absorption of a nuclear particle in a nucleus during a nuclear reaction is analogous to the process of condensation from the vapour to the liquid phase in case of liquid drop.

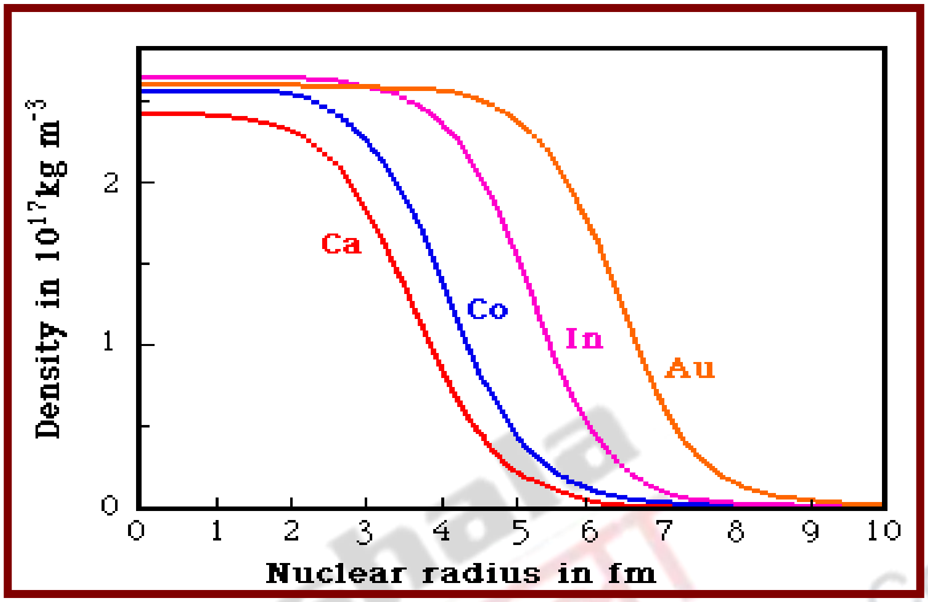


Fig. 2: variation of mass density of nuclear matter with nuclear radius is nearly constant.

The liquid drop model is not very successful in describing the low lying excited states of the nucleus. Because of the collective motions of the large number of nucleons involved, the model gives rise to closely spaced energy level. Actually however, these are found to be quite widely spaced at low excitation energies.

3. Bethe- Weizsacker Formula

The dependence of nuclear masses as a function of A and Z was first introduced in 1935 by German physicist Carl Friedrich von Weizsäcker, which is known as the Weizsäcker formula or the semi-empirical mass formula.

This semi-empirical formula for the nuclear masses or (nuclear binding energies) gives a connection between the theory of nuclear matter with experimental information and is based on the liquid drop model of nucleus. This formula describes the binding energy B (A, Z) of spherical nuclei with mass number A, atomic number Z, and neutron number N and has the following form of binding energy.

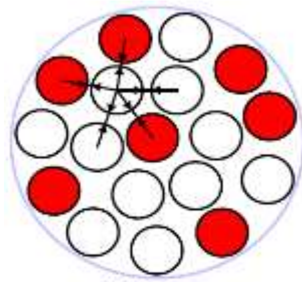
$$B(A,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(N-Z)^2}{A} \pm \delta + \eta$$

Volume term — Surface term — Coulomb term — Asymmetry term — Pairing term

The explanation of each term of the above equation is given as follows

(1) Volume Energy Term- $a_V A$**:** The first term in the semi-empirical mass formula is the volume energy term that describes how the binding energy is mostly proportional to A, and accounts for the binding energy of all the nucleons as if everyone entirely surrounded by other nucleons. Where a_V is a constant that describes the nuclear force in infinite nuclear matter, to be derived from fits to experimental data.

Since nucleons are closely packed in the nucleus and the nuclear force has a very short range, each individual nucleon interacts only with its nearest neighbors. This means that independently of the total number of nucleons, each one of them contribute in the same way. Thus the force is not proportional to A (A − 1)/2 ∼ A^2 but it is proportional to A.



(2) Surface Energy Term - $a_S A^{2/3}$ **:** The second term is the surface energy term, also based on the strong force, is a correction term to the volume term as not all the nucleons are surrounded by other nucleons but lie in or near the surface. A nucleon at the surface of a nucleus interacts with less number of nucleons than one in the interior of the nucleus, so its binding energy is less. So, a term proportional to the number of nucleons in the surface region must be subtracted from the volume term. This giving rise to a surface term, similar to the surface tension in liquids.

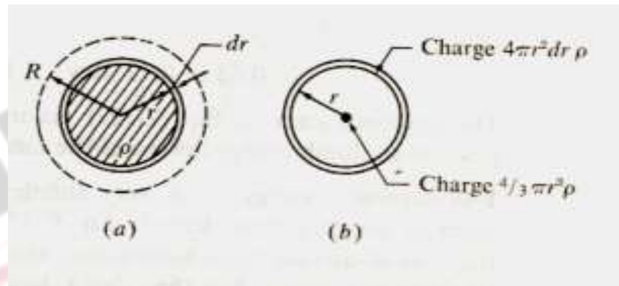
The surface energy loss **E_S** for a sphere with surface tension constant σ can be written as

$$\mathbf{E_s = -\sigma\, 4\pi R^2 = -\sigma\, 4\pi r^2 (A^{1/3})^2 = -\, a_S\, A^{2/3}}$$

Surface

(3) Coulomb energy term- $a_C \frac{Z^2}{A^{1/3}}$ **:** The third term in the semi-empirical mass formula is the Coulomb energy term that arises due to the electrostatic repulsion between protons. Since protons carry positive electric charge, they mutually repel one another in the nucleus. This effect produces an additional binding energy loss that requires inclusion of a Coulomb term Ec.

$$V_{Coulomb} = \int_0^R \left(\frac{4}{3}\pi r^3 \rho\right) \cdot (4\pi r^2 dr \rho) \cdot \frac{1}{r}$$
$$= \frac{16}{15}\pi^2 \rho^2 R^5 = \frac{3}{5}\frac{Z^2 e^2}{R}$$



According to Coulomb's Law, the electrostatic potential energy of a uniformly charged sphere is given as

$E_C = -V_{Coulomb}$

$E_C = -(3/5)Z^2e^2/R$, or, substituting for $R= A^{1/3}$, we get

$E_C = -\ a_C\ Z^2/A^{1/3}$

(4) Asymmetry energy term- $a_A \frac{(N-Z)^2}{A}$ **:** This term account for the fact that in the light nuclei there is a tendency for the number of protons and neutron to be equal (N=Z) for stable configuration. As we move to the heavier nuclei, there is increase in number of protons which tend to decrease the binding energy due to Coulomb repulsion between them. To compensate this Coulomb repulsion some extra neutrons must be present to provide addition n-n bonds. However, this disturbs the condition of equality of Z and N to form the most stable configuration in the absence of Coulomb effect. Thus due to this asymmetry in the neutron-proton numbers there will be a reduction in the binding energy by an amount proportional to $(N - Z)^2$.

Since $N - Z= A - 2Z$, we can write the asymmetry energy as

$$Ea = a_A \frac{(A-2Z)^2}{A}$$

(5) Pairing energy term - $\pm\delta$ **:** The "*pairing term*" accounts for the fact that a pair of like nucleons is more strongly bound than pair of unlike nucleons. Then it means that the binding energy is greater ($\delta > 0$) if we have an even-even nucleus, where all the neutrons and all the protons are paired-off. If we have a nucleus with both an odd number of neutrons and of protons, it is thus favorable to convert one of the protons into a neutrons or vice-versa. Thus, with all other factor constant, we have to subtract ($\delta < 0$) a term from the binding energy for odd-odd configurations. For even-odd configurations there is not any influence from this pairing energy (δ=0). The pairing term for different nuclei are then given as

- For odd *A* nuclei (*Z* even, *N* odd or *Z* odd, *N* even) → **δ = 0**.
- For even *A* there are two cases : **(*a*). *Z* odd, *N* odd (oo) → – δ**

(*b*). *Z* even, *N* even (ee) → + δ

$$\delta\,(Z,A) = \frac{a_P}{A^{1/2}}, \quad a_P = 12\,\text{MeV}$$

(6) Shall Correction Energy Term – η: This term accounts for nuclear shell effect when *Z* or *N* is a magic number. This term is much less important than other terms.

The values of constant is obtained by best fitting the curve and has the following values

a_V = 15.560 MeV , a_S = 17.230MeV , a_C = 0.6970 MeV ,

a_A = 23.385 MeV , a_P = 12.000 MeV

The relative contributions of each of the term to the binding energy per nucleon versus mass number A is shown in figure 3.

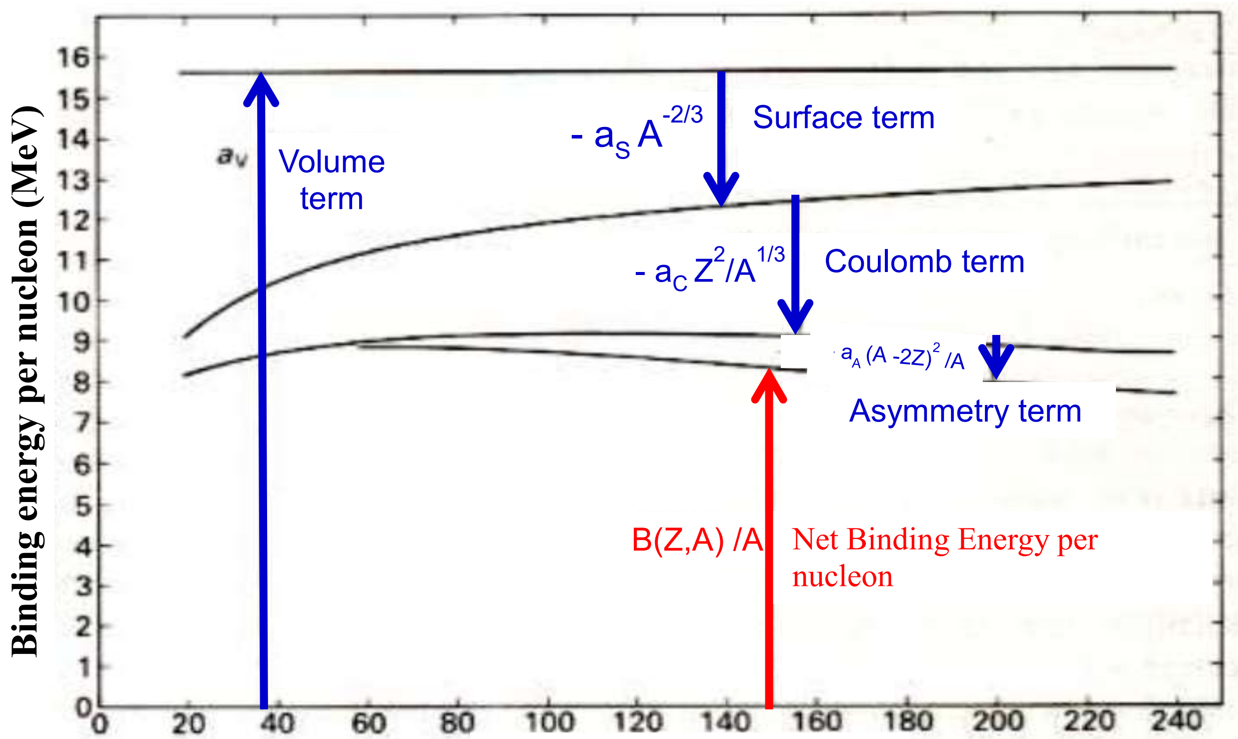


Fig. 3: Contribution of the volume, surface, Coulomb, and asymmetry energies to the total average binding energy as a function of mass number.

In the above fig. 3 the horizontal line at ~ 16 MeV represents the contribution of the volume energy. This is reduced by the surface energy, the asymmetry energy and the Coulomb energy to the effective binding energy of ~8 MeV (*lower line*). The contributions of the asymmetry and Coulomb terms increase rapidly with *A*, while the contribution of the surface term decreases.

Summary

Nuclei have small dimensions, so they are not accessible directly. Because of the difficulties in developing a satisfactory theory of nuclear structure, different nuclear models have been proposed to explain the observed nuclear properties of different nuclei. Different models are based on the different approach and analogy. The usefulness of a model depends on the extent to which its predications are confirmed by the experiments. The liquid drop model is based on the analogy of nucleus with the liquid drop, and it gives a simple explanation for the nuclear binding energy curve.